Pebble Games for Linguistic Anaphora

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Outline

A (very) short introduction to linguistics, categorically

Some of the many branches of linguistics:

- Phonetics, phonology, morphology (subword level)
- Syntax, semantics, pragmatics (sentence level)
- Text linguistics, dialogue and discourse analysis
- Psycho-, neuro-, socio-, historical linguistics
- Computational linguistics, natural language processing (NLP)

Syntax from free monoidal categories

Definition

An *unrestricted grammar* is a tuple $G = (V, X, R, s)$ where:

- V and X are finite sets of terminal and non-terminal symbols,
- $R \subseteq (V + X)^+ \times (V + X)^\star$ is a finite set of rewrite rules,[1]
- $s \in X$ is the sentence type (or start symbol).

Proposition

A string $w_1, \dots, w_n \in V^\star$ is grammatical in G iff there is an arrow $g : s \to w_1 \otimes \dots \otimes w_n$ in $\mathbf{G}$, the free monoidal category with generating objects $V + X$ and arrows R.

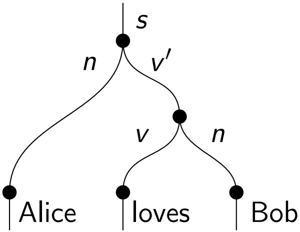
Theorem (Post, Markov 1947)

Unrestricted grammars (i.e. the word problem for semigroups) are undecidable.

[1] $+$ is disjoint union and $\times$ is Cartesian product, X^+ and $X^\star$ are the free semigroup and monoid respectively.

Syntax from free monoidal categories: example

$$V = \{ \text{Alice}, \text{loves}, \text{Bob} \}$$
$$X = \{ s, n, v, v' \}$$
$$R = \{s \to n \otimes v', \; v' \to v \otimes n, \; n \to \text{Alice}, \; v \to \text{loves}, \; n \to \text{Bob}\}$$



Syntax from free rigid monoidal categories

Definition

A *pregroup grammar* is a tuple $G = (V, B, D, s)$ where V and B are finite sets with $s \in B$ and $D \subseteq V \times (B \times \mathbb{Z})^\star$ is a dictionnary.

Definition

A monoidal category $\mathbf{C}$ is *rigid* when every object x has left and right adjoints x^l and x^r and four morphisms $x \otimes x^l \xrightarrow{\epsilon} 1 \xrightarrow{\eta} x^l \otimes x$ and $x^r \otimes x \xrightarrow{\epsilon'} 1 \xrightarrow{\eta'} x \otimes x^r$ called *cups and caps*, subject to $(\epsilon' \otimes 1_x) \circ (1_x \otimes \eta') = 1_x = (1_x \otimes \epsilon) \circ (\eta \otimes 1_x)$.

Proposition

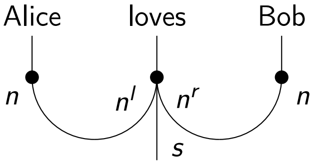
A string $w_1, \dots, w_n \in V^\star$ is grammatical in G iff there is a morphism $g : w_1 \otimes \dots \otimes w_n \to s$ in $\mathbf{G}$ the free rigid monoidal category with generating objects $V + B$ and arrows D.

Theorem (Buszkowski, Moroz 2008)

Pregroup grammars are context-free, they are efficiently parsable.

Syntax from free rigid monoidal categories: example

$V = \{ \text{Alice}, \text{loves}, \text{Bob} \}$
$B = \{ s, n \}$
$D = \{ \text{Alice} \to n, \ \ \text{loves} \to n^r \otimes s \otimes n^l, \ \ \text{Bob} \to n \}$



Semantics as monoidal functors

Frege's principle of compositionality: the meaning of a complex expression is determined by the meanings of its constituent expressions and the rules used to combine them.
Categorically: the meaning of a grammatical sentence $g : w_1 \otimes \cdots \otimes w_n \to s$ is given by its image $F(g)$ under a monoidal functor $F : \mathbf{G} \to \mathbf{S}$ for $\mathbf{S}$ a monoidal category.

Example

Montague semantics is a closed functor $\mathbf{G} \to \mathbf{Set}$ for $\mathbf{G}$ a categorial grammar (i.e. a free biclosed category). The meaning of words are given by lambda terms, the meaning for a sentence is a closed logical formula.

Example

Distributional Compositional (DisCo) models are rigid functors $\mathbf{G} \to \mathbf{Vect}$ for $\mathbf{G}$ a pregroup grammar. The meaning of words are given by tensors, the meaning for a sentence is a scalar.

Pragmatics with language-games

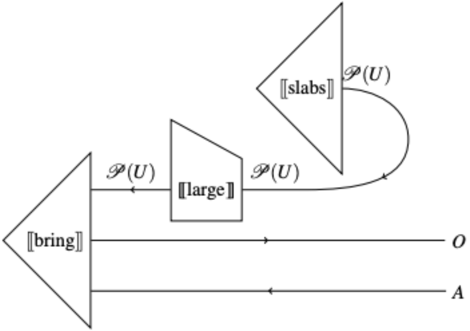
Observation: meaning depends on context. Wittgenstein's language-games: "asking, thanking, cursing, greeting, praying". The same utterance "Water!" can be a request to a waiter, the answer to a question or the lyrics of a song.

Example

«The language is meant to serve for communication between a builder A and an assistant B. A is building with building-stones: there are blocks, pillars, slabs and beams. B has to pass the stones, in the order in which A needs them. For this purpose they use a language consisting of the words "block", "pillar", "slab", "beam". A calls them out; — B brings the stone which he has learnt to bring at such-and-such a call.
Conceive this as a complete primitive language. »
Wittgenstein, *Philosophical Investigations* (1953)

Pragmatics with language-games, categorically

Language-games as teleological functors $\mathbf{G} \to \mathbf{OG}$ (J. Hedges and M. Lewis) for **OG** the category of open games.



Language-games as rigid functors $\mathbf{G} \to \mathcal{A}(\mathbf{OG})$ for $\mathcal{A} : \mathbf{MonCat} \to \mathbf{RigidCat}$ the free rigid completion (joint work with G. De Felice, E. Di Lavore and M. Roman).

Anaphora resolution and question-answering

Anaphora: the use of an expression whose interpretation depends upon another expression in context (its antecedent).

Anaphora resolution: given a piece of text, assign each anaphora to its antecedent. One of the key challenges of NLP.

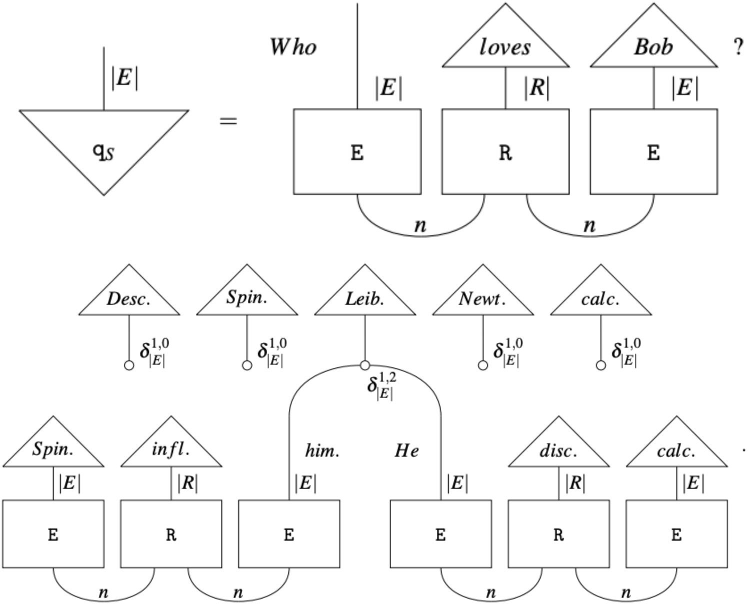
Question-answering: a game between (Zen) master and student.

Previous work with G. De Felice and K. Meichanetzidis:
given a question and a corpus with its anaphora resolution,
question-answering is NP-complete for relational models.

Example

Donkey sentences: "When a farmer owns a donkey, he beats it."
"he" $\mapsto$ "farmer", "it" $\mapsto$ "donkey"
Q: Who gets beaten? A: The donkey.

Anaphora resolution and question-answering: example



Relational models and conjunctive queries

Theorem (Bonchi, Seeber and Sobocinski (2018))

Conjunctive queries over a relational signature Σ *are the arrows of the free Cartesian bicategory* $\mathbf{CB}(\Sigma)$.
Preorder enrichment captures conjunctive query containment.
CB morphisms $K : \mathbf{CB}(\Sigma) \to \mathbf{Rel}$ *are relational models, i.e. they define a universe* $U = K(1)$ *and an interpretation* $K(R) \subseteq U^{ar(R)}$ *for each relational symbol* $R \in \Sigma$.

Proposition

For a pregroup grammar G*, the rigid functors* $F : \mathbf{G} \to \mathbf{Rel}$ *factor as* $F = K \circ L$ *for* $L : \mathbf{G} \to \mathbf{CB}(\Sigma)$ *a rigid functor and* $K : \mathbf{CB}(\Sigma) \to \mathbf{Rel}$ *a relational model.*

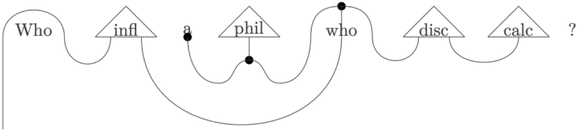
Corollary

Semantics, entailment and question-answering are NP-complete.

Relational models and conjunctive queries: example

Example 4.3. We take $L : \mathbf{G} \to \mathbf{CB}(\Sigma)$ to map the question word "Who" to the compact-closed structure, the determinant "a" to the unit and the common noun "philosopher" to the symbol $phil \in \Sigma$ composed with the comonoid. We can now find the nouns that answer the question $r \in \mathbf{G}(u, q)$ of example 1.11 as the evaluation of the following query:

$L(r) \quad =$



$$\Lambda(L(r)) \quad = \quad \exists\, x_1 \,\exists\, x_2 \;\cdot\; infl(x_0, x_1) \;\wedge\; phil(x_1) \;\wedge\; disc(x_1, x_2) \;\wedge\; calc(x_2)$$

If "Spinoza influenced the philosopher Leibniz" and "Leibniz discovered calculus" are in the corpus C, we have $L(Spinoza \to n) \in \texttt{QuestionAnswering}(C, \mu, \Lambda(L(r)))$.

Anaphora, pebbles and the magical number 7 ± 2

Q: How can we find a tractable fragment for question-answering?
A: Bounded tree-width!

Theorem (Abramsky, Dawar and Wang (2017))

The tree-width k of a relational model is its coalgebra number for the pebble game comonad, as well as its number of variables.

Proposition (Miller (1956))

Experimentally, $k = 7 \pm 2$.

Q: Can we use this to implement LSTM-type neural networks functorially?
Q: Can we use other game comonads to model computational resources in language?
Q: Would they qualify as Wittgensteinian language-games?

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